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ZERO DELAMINATION OF SMART COMPOSITE LAMINATED ANGLE PLY USING HSDT WITH ZIG ZAG FUNCTION

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ABSTRACT

Thermal analysis of smart composite laminated plate using higher order theory with zig-zag functions. To develop the analytical procedure and to investigate the thermal characteristics, the material is considered to be orthotropic under thermal load based on higher order displacement model with zig-zag function, without enforcing zero transverse shear stress on the top and bottom faces of the laminated plates. The related functions and derivations for equation of motion are obtained using the dynamic version of the principle of virtual work or Hamilton's principle. The solutions are obtained by using Navier's and numerical methods for anti-symmetric angle-ply with specific type of simply supported boundary conditions SS. Computer programs have been developed to find the stresses and deflections for various aspect ratios, side thickness ratios (a/h) and voltages.

KEYWORDS: C++, Hamilton's principle, HSDT-higher order shear deformation theory, Navier's Strokes equation, SS- simply supported.

INTRODUCTION

In this work actuator is coupled to the top of the laminated composite material plates to achieve its thermal characteristics, which are tabulated in non-dimensional form of various aspect ratios, side thickness ratios (a/h) and voltages, and also to evaluate electric potential function by solving higher order differential equation satisfying electric boundary conditions along the thickness direction of piezoelectric layer. These solutions are plotted as a function of aspect ratio, thickness ratio (z/h) and voltages etc. The effects of shear deformation, coupling, degree of orthotropy and voltage on the response of smart composite laminated plates are investigated.

FORMULATION OF HSDT WITH ZIG- ZAG FUNCTION

A laminated plate of $0 \le x \le a$; $0 \le y \le b$ and $-h/2 \le z \le h/2$ is considered.

The displacement components u (x, y, z, t), v (x, y, z, t) and w (x, y, z, t) at any point in the plate are expanded in terms of the thickness coordinate. In this work the in plane displacements are expanded as cubic functions of the thickness coordinate. The displacement field which assumes w (x, y, z, and t) constant through the plate thickness is expressed as:

$$\left.\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) + z^2 u_o^*(x,y) + z^3\theta_x^*(x,y) + \theta_k s_1(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) + z^2 v_o^*(x,y) + z^3\theta_y^*(x,y) + \theta_k s_2(x,y) \\ w(x,y,z) &= w_o(x,y) \end{aligned}\right\} \qquad \ldots\ldots\ldots..\ 1$$

Where

uo, v_o, w_o , s_1 and s_2 denote the displacements of a point (x, y) at the midplane.

θ_x, θ_y are rotations about y and x–axes at midplane.

u_o*, v_o^*, θ_x^*, θ_y^* are the corresponding higher–order deformation terms defined at the midplane.

θ_k is the Zig-Zag function, defined as:

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$$\theta_k = 2(-1)^k \frac{Z_k}{h_k}$$

z_k is the local transverse coordinate with its origin at the centre of the k^{th} layer and h_k is the corresponding layer thickness.

The Zig-Zag function is piecewise linear with values of –1 and 1 alternately at the different interfaces.

The strain components are:

$$\varepsilon_x = \varepsilon_{x0} + zk_{sx} + z^2\varepsilon^*_{x0} + z^3k^*_x$$
$$\varepsilon_y = \varepsilon_{y0} + zk_{sy} + z^2\varepsilon^*_{y0} + z^3k^*_y$$
$$\varepsilon_z = 0$$
$$\gamma_{xy} = \varepsilon_{xy0} + zk_{sxy} + z^2\varepsilon^*_{xy0} + z^3k^*_{xy}$$
$$\gamma_{yz} = \phi_{sy} + z\varepsilon_{yz0} + z^2\phi^*_y$$
$$\gamma_{xz} = \phi_{sx} + z\varepsilon_{xz0} + z^2\phi^*_x$$

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LAMINATE CONSTITUTIVE EQUATIONS

$$\{\sigma\} = [C]\{\varepsilon\} - [e]\{E\}$$

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As per equation 3

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^L = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{12} & Q_{22} & Q_{23} \\ Q_{13} & Q_{23} & Q_{33} \end{bmatrix}^L \begin{Bmatrix} \epsilon_x - \alpha_x\Delta T \\ \epsilon_y - \alpha_y\Delta T \\ \gamma_{xy} - \alpha_{xy}\Delta T \end{Bmatrix}^L - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \frac{-\partial\xi(x,y,z)}{\partial x} \\ \frac{-\partial\xi(x,y,z)}{\partial y} \\ \frac{-\partial\xi(x,y,z)}{\partial z} \end{Bmatrix}$$

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$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^L = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix}^L \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^L - \begin{bmatrix} 0 & e_{24} & 0 \\ e_{15} & 0 & 0 \end{bmatrix} \begin{Bmatrix} \frac{-\partial\xi(x,y,z)}{\partial x} \\ \frac{-\partial\xi(x,y,z)}{\partial y} \\ \frac{-\partial\xi(x,y,z)}{\partial z} \end{Bmatrix}$$

$$E_x = \frac{-\partial\xi(x,y,z)}{\partial x} \quad E_y = \frac{-\partial\xi(x,y,z)}{\partial y} \quad E_z = \frac{-\partial\xi(x,y,z)}{\partial z}$$

$\xi(x, y, z)$ is the electro static potential

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The superscript t denotes the transpose of a matrix.

$Q_{11} = C_{11} C^4 + 2 (C_{12} + 2 C_{33}) S^2 C^2 + C_{22} S$

$Q_{12} = C_{12} (C^4 + S^4) + (C_{11} + C_{22} - 4 C_{33}) S^2 C^2$

- $Q_{13} = (C_{11} - C_{12} - 2 C_{33}) S C^3 + (C_{12} - C_{22} + 2 C_{33}) C S^3$
- $Q_{22} = C_{11} S^4 + C_{22} C^4 + (2 C_{12} + 4 C_{33}) S^2 C^2$
- $Q_{23} = (C_{11} – C_{12} – 2 C_{33}) S^3C + (C_{12} – C_{22} +2 C_{33})C^3 S$
- $Q_{33} = (C_{11} – 2 C_{12} + C_{22} – 2 C_{33}) S^2 C^2 + C_{33} (C^4+S^4)$
- $Q_{44} = C_{44} C^2 + C_{55} S^2$
- $Q_{45} = (C_{55} – C_{44}) C S$
- $Q_{55} = (C_{44} S^2 + C_{55} C^2)$

$\alpha_x = \alpha_1 C^2 + \alpha_2 S^2$, $\alpha_y = \alpha_2 C^2 + \alpha_1 S^2$, $\alpha_{xy}= (\alpha_1 - \alpha_2) SC$

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The stress strain relationship in global X – Y – Z coordinate is written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 \\ Q_{21} & Q_{22} & Q_{23} & 0 & 0 \\ Q_{31} & Q_{32} & Q_{33} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & Q_{45} \\ 0 & 0 & 0 & Q_{54} & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}$$

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The governing equations of displacement model are derived using the principle of virtual work or Hamilton's principle.

$$\int_0^T (\delta U + \delta V - \delta K)\, dt = 0$$

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The virtual work statement shown in Eq (7), integrating through the thickness of laminate, the in-plane and transverse force and moment resultant relations in the form of matrix obtained as:

$$\begin{Bmatrix} N \\ N^* \\ -- \\ M \\ M^* \\ -- \\ Q \\ Q^* \end{Bmatrix} = \begin{bmatrix} A & | & B & | & 0 \\ \overline{B^t} & | & \overline{D_b} & | & \overline{0} \\ \overline{0} & | & \overline{0} & | & \overline{D_s} \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \varepsilon_0^* \\ -- \\ K_s \\ K^* \\ -- \\ \phi_s \\ \phi^* \end{Bmatrix} - \begin{Bmatrix} N^T \\ N^{*T} \\ -- \\ M^T \\ M^{*T} \\ -- \\ 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} N^{PZ} \\ N^{*PZ} \\ -- \\ M^{PZ} \\ M^{*PZ} \\ -- \\ Q^{PZ} \\ Q^{*PZ} \end{Bmatrix}$$

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Using Hamilton's principle for total potential energy and by Equating the coefficients of each of virtual displacements δu_0, δv_0, δw_0, $\delta\theta_x$, $\delta\theta_y$, δu_0^*, δv_0^*, $\delta\theta_x^*$, $\delta\theta_y^*$, δs_1, δs_2 to zero, the following equations of motion are obtained:

$$\delta u_0 : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_1\ddot{u}_0 + I_2(\ddot{\theta}_x + R\ddot{s}_1) + I_3\ddot{u}_0^* + I_4\ddot{\theta}_x^*$$

$$\delta v_0 : \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = I_1\ddot{v}_0 + I_2(\ddot{\theta}_y + R\ddot{s}_2) + I_3\ddot{v}_0^* + I_4\ddot{\theta}_y^*$$

$$\delta w_0 : \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = I_1\ddot{w}_0$$

$$\delta\theta_x : \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = I_2\ddot{u}_0 + I_3(\ddot{\theta}_x + R\ddot{s}_1) + I_4\ddot{u}_0^* + I_5\ddot{\theta}_x^*$$

$$\delta\theta_y : \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = I_2\ddot{v}_0 + I_3(\ddot{\theta}_y + R\ddot{s}_2) + I_4\ddot{v}_0^* + I_5\ddot{\theta}_y^*$$

$$\delta u_0^* : \frac{\partial N_x^*}{\partial x} + \frac{\partial N_{xy}^*}{\partial y} = I_3\ddot{u}_0 + I_4(\ddot{\theta}_x + R\ddot{s}_1) + I_5\ddot{u}_0^* + I_6\ddot{\theta}_x^*$$

$$\delta v_0^* : \frac{\partial N_y^*}{\partial y} + \frac{\partial N_{xy}^*}{\partial x} = I_3\ddot{v}_0 + I_4(\ddot{\theta}_y + R\ddot{s}_2) + I_5\ddot{v}_0^* + I_6\ddot{\theta}_y^*$$

$$\delta\theta_x^* : \frac{\partial M_x^*}{\partial x} + \frac{\partial M_{xy}^*}{\partial y} - Q_x^* = I_4\ddot{u}_0 + I_5(\ddot{\theta}_x + R\ddot{s}_1) + I_6\ddot{u}_0^* + I_7\ddot{\theta}_x^*$$

$$\delta\theta_y^* : \frac{\partial M_y^*}{\partial y} + \frac{\partial M_{xy}^*}{\partial x} - Q_y^* = I_4\ddot{v}_0 + I_5(\ddot{\theta}_y + R\ddot{s}_2) + I_6\ddot{v}_0^* + I_7\ddot{\theta}_y^*$$

$$\delta s_1 : \frac{\partial RM_x}{\partial x} + \frac{\partial RM_{xy}}{\partial y} - R\,Q_x = I_2R\ddot{u}_0 + I_3R(\ddot{\theta}_x + R\ddot{s}_1) + I_4R\ddot{u}_0^* + I_5R\,\ddot{\theta}_x^*$$

$$\delta s_2 : \frac{\partial RM_y}{\partial y} + \frac{\partial RM_{xy}}{\partial x} - R\,Q_y = I_2R\ddot{v}_0 + I_3R(\ddot{\theta}_x + R\ddot{s}_2) + I_4R\ddot{v}_0^* + I_5R\,\ddot{\theta}_y^*$$

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The Navier's solutions of simply supported anti symmetric angle ply laminated plates.
The SS boundary conditions for the anti-symmetric angle ply laminated plates are:

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At edges x = 0 and x = a

u_0=0,w_o=0, θ_y=0, N_{xy}=0, M_x=0, u_0^*=0, θ_y^*=0, M_x^*=0, N_{xy}^*=0, S_1 =0 … 10.1

At edges y = 0 and y = b

v_0=0,w_o=0, θ_x=0, N_{xy}=0, M_y =0, v_0^*=0, θ_x^*=0, M_y^*=0, N_{xy}^* =0, S_2 =0 ....
10.2

The displacements at the mid plane will be defined to satisfy the boundary conditions in Eq. (10). These displacements will be substituted in governing equations to obtain the equations in terms of A, B, D parameters. The obtained equations will be solved to find the behavior of the laminated composite plates.

RESULTS AND DISCUSSIONS

The material properties used for each orthotropic layer are
Graphite Epoxy:

E_1/E_2= 2.5 G_{12}/E_2= 0.5 G_{23}/E_2= 0.2 E_2=E_3= 10^2 N/ m^2 μ_{12}= μ_{23}=μ_{13}=0.25
α_1 =1 × 10^{-6} $^0C^{-1}$, α_2 =1.125 × 10^{-3} $^0C^{-1}$, α_3 =1.125 × 10^{-3} $^0C^{-1}$

PFRC Layer:
C_{11} = 32.6Gpa, C_{12} = C_{21} = 4.3Gpa, C_{13} = C_{31} = 4.76Gpa, C_{22} = C_{23} =7.2Gpa, C_{23} = 3.85Gpa, C_{44} =1.05Gpa, C_{55} = C_{66} =1.29Gpa, e_{31}= -6.76C/m^2, g_{11}=g_{22} = 0.037E-9C/V m, g_{33}=10.64E - 9 C/V m

The center deflections and stresses are presented here in non-dimensional form using the following multipliers:
$m_6 = e_2\alpha_1 t_0$, $m_7 = \frac{a_1^2 t\, \alpha_1 t_0}{t^2}$

The variation of non-dimensionalized transverse deflection (w) against side thickness ratios as a function of number of layers at voltage 100V for a simply supported angle ply smart composite laminated plates is showed in figure1. From the figure 1 it is noted that as the side to thickness ratio is increasing the transverse deflection is decreasing and is observed maximum for 2 layers with an error of 1.2 percent. If the coupling coefficients increase in magnitude, hence the effect of coupling increases with the increase in modulus ratio for transverse deflections (w). Figure 2 shows the variation of non-dimensionalized normal stress (σ_x) against thickness coordinates as a function of number of layers at voltage 100V for a simply supported angle ply smart composite laminated plates. From plot it is noted that, normal stress is observed maximum for 2 layers with 0.6 pecent error and the stresses are tending to be maximum for smaller thickness coordinate ratios. Figure 3 shows the variation of non-dimensionalized transverse shear stress (τ_{xz}) against side thickness ratios as a function of number of layers at voltage 100V for a simply supported angle ply smart composite laminated plates. From plot 3 it is observed, transverse shear stress is maximum for 2 layers with 1.2 percent as error. As side to thickness ratio is increasing transverse shear stress is gradually decreasing.

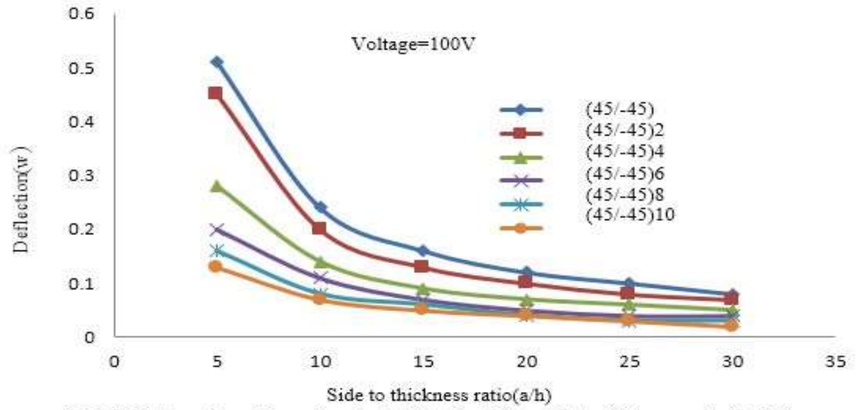

Fig.1.Variation of non dimensionalised deflection (w) vs side to thickness ratio (a/h) for a simply supported angle ply smart composite laminated plate.

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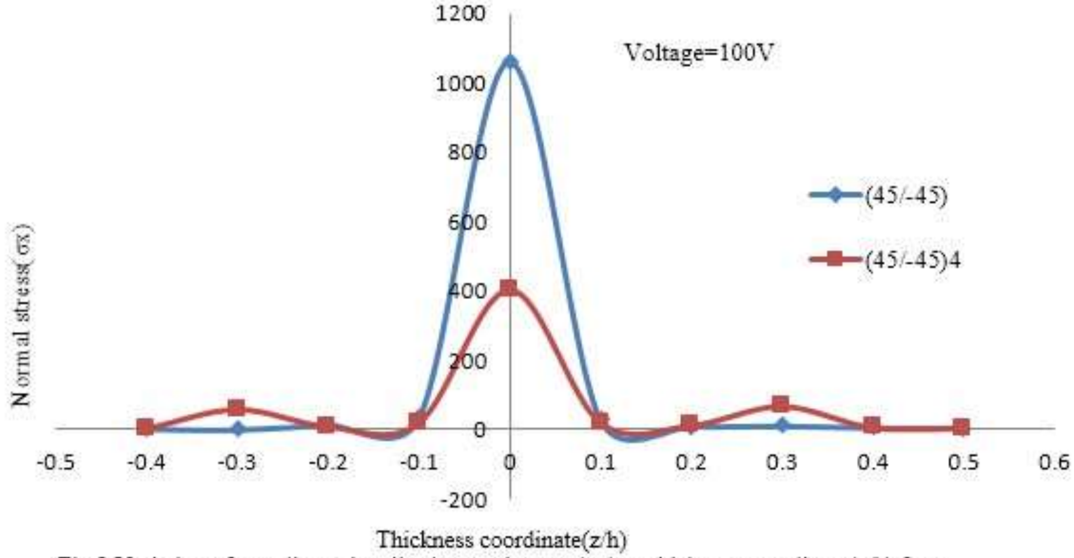

Fig.2.Variation of non dimensionalised normal stress (σx) vs thickness coordinate(z/h) for a simply supported angle ply smart composite laminated plate.

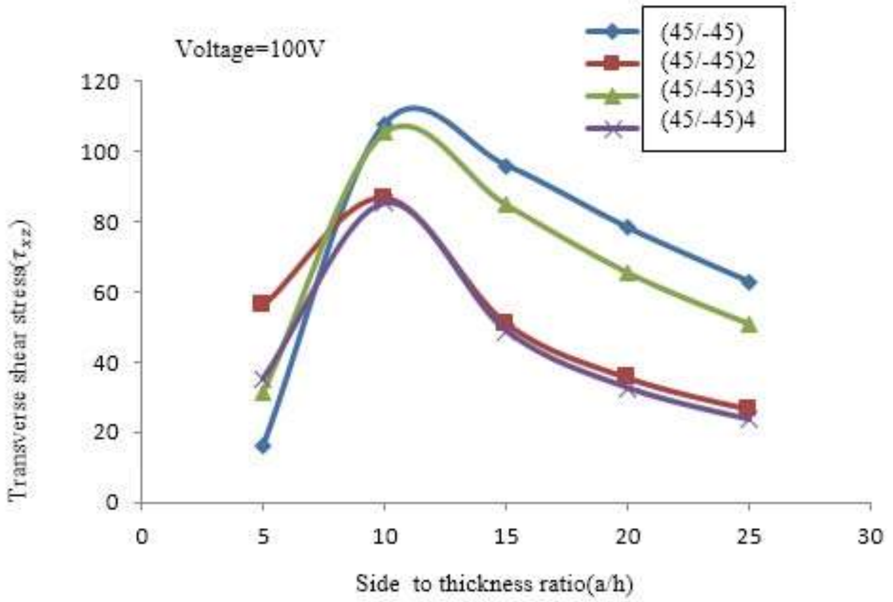

Fig.3.Non dimensionalised transverse shear stress(τ_{xz}) vs side to thickness ratio for a simply supported angle ply smart composite laminated plates.

CONCLUSION

Analytical procedure is developed in this paper for thermal analysis of smart composite laminated plates subjected to electromechanical loading. The non-dimensional transverse displacement and shear stresses are obtained for various voltages, aspect ratios and thickness coordinates. It is concluded from the results that, the transverse displacement varies through the thickness non linearly for plate subjected to temperature gradient than for those subjected to mechanical loads. The zig-zag function is a valuable tool to enhance the performance of both classical and advanced theories.

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